

# LC Circuits:

**L** inductor (solenoid)

**C** capacitor

**Electrical oscillations:** An electrical circuit containing an inductor and a capacitor can be stimulated to oscillate, just like a pendulum or a mass on a spring: The current flows one way (for example from the capacitor to the inductor), then reverses its direction and flows back the other way. Both the current in the circuit and the charge in the capacitor depend on time in a sinusoidal fashion.

$$Q(t) = Q_0 \cos(\omega_0 t + f) \qquad I(t) = -\frac{dQ}{dt} = I_0 \sin(\omega_0 t + f)$$

**natural angular frequency:**  $\omega_0 = \frac{1}{\sqrt{LC}}$

The **energy** in the circuit is first stored in the charged capacitor as **electrostatic energy**, then the energy is converted into **magnetic energy** stored in the inductor. Then the energy flows back into the capacitor, and so on. The **total energy** (sum of potential energies in the capacitor and the inductor) remains constant.

## **Mechanical Oscillations:**

$$x \quad v \quad m \quad \frac{1}{2}mv^2 \quad k \quad \frac{1}{2}kx^2 \quad F \quad P = Fv$$

## **Electrical Oscillations:**

$$Q \quad I \quad L \quad \frac{1}{2}LI^2 \quad \frac{1}{C} \quad \frac{1}{2}\frac{Q^2}{C} \quad V \quad P = VI$$

# LRC Circuits:

**L** inductor (solenoid)      **R** resistor      **C** capacitor

When a resistor is introduced into an LC circuit, the oscillations are damped. These oscillations are similar to damped mechanical oscillations (friction). There are three cases, depending on the size of the resistor:

1. Underdamped case:  $R < 2\omega_0 L$

The system still oscillates, but the oscillations are damped with a frequency

$$\omega' = \sqrt{\omega_0^2 - \left(\frac{R}{2L}\right)^2}$$

smaller than the natural frequency in the undamped case.

$$Q(t) = Q_0 e^{-\frac{Rt}{2L}} \cos(\omega' t + f)$$

2. Critical damping:  $R = 2\omega_0 L$

No oscillations. The charge and current oscillations go to zero in the shortest time.

3. Overdamped case:  $R > 2\omega_0 L$

No oscillations.